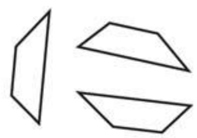
4.2 Apply Congruence and Triangles

Before	You identified congruent angles.
Now	You will identify congruent figures.
Why?	So you can determine if shapes are identical, as in Example 3.

Key Vocabulary
- **congruent figures**
- **corresponding parts**

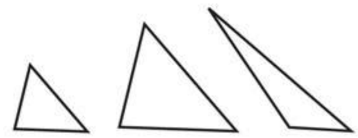
Two geometric figures are *congruent* if they have exactly the same size and shape. Imagine cutting out one of the congruent figures. You could then position the cut-out figure so that it fits perfectly onto the other figure.

Congruent



Same size and shape

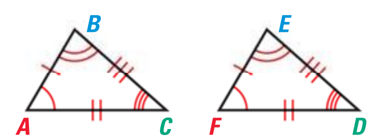
Not congruent



Different sizes or shapes

In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

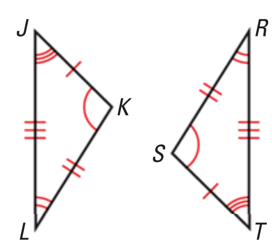
CONGRUENCE STATEMENTS When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles at the right are $\triangle ABC \cong \triangle FED$ or $\triangle BCA \cong \triangle EDF$.



Corresponding angles	$\angle A \cong \angle F$	$\angle B \cong \angle E$	$\angle C \cong \angle D$
Corresponding sides	$\overline{AB} \cong \overline{FE}$	$\overline{BC} \cong \overline{ED}$	$\overline{AC} \cong \overline{FD}$

EXAMPLE 1 Identify congruent parts

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



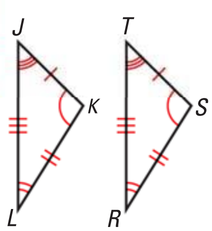
Solution

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

VISUAL REASONING
To help you identify corresponding parts, turn $\triangle RST$.

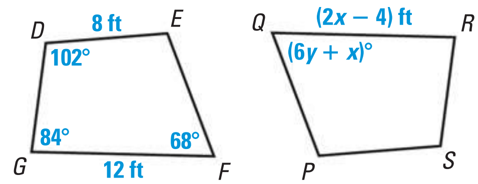


EXAMPLE 2 Use properties of congruent figures

In the diagram, *DEFG* ≅ *SPQR*.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know that $\overline{FG} \cong \overline{QR}$.

$FG = QR$

$12 = 2x - 4$

$16 = 2x$

$8 = x$

b. You know that $\angle F \cong \angle Q$.

$m\angle F = m\angle Q$

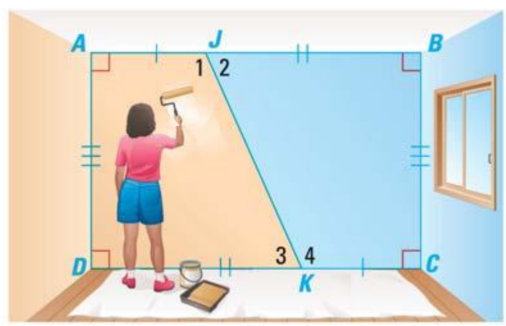
$68° = (6y + x)°$

$68 = 6y + 8$

$10 = y$

EXAMPLE 3 Show that figures are congruent

PAINTING If you divide the wall into orange and blue sections along $\overline{JK}$, will the sections of the wall be the same size and shape? *Explain*.



Solution

From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{AB} \parallel \overline{DC}$. Then, $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

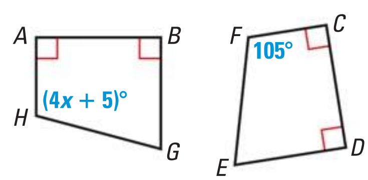
The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property, $\overline{JK} \cong \overline{KJ}$. All corresponding parts are congruent, so $AJKD \cong CKJB$.

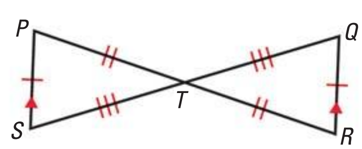
▶ Yes, the two sections will be the same size and shape.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

In the diagram at the right, *ABGH* ≅ *CDEF*.

1. Identify all pairs of congruent corresponding parts.
2. Find the value of x and find $m\angle H$.
3. Show that $\triangle PTS \cong \triangle RTQ$.



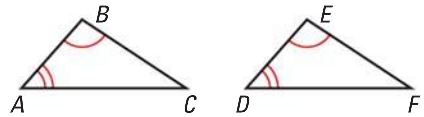


THEOREM *For Your Notebook*

THEOREM 4.3 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

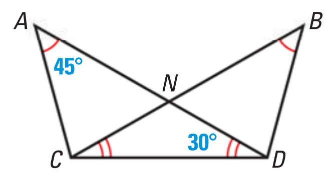
Proof: Ex. 28, p. 230



If $\angle A \cong \angle D$, and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Use the Third Angles Theorem

Find $m\angle BDC$.



Solution

$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

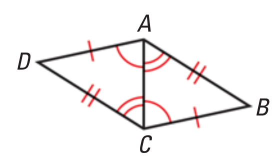
▶ So, $m\angle ACD = m\angle BDC = 105°$ by the definition of congruent angles.

ANOTHER WAY

For an alternative method for solving the problem in Example 4, turn to page 232 for the **Problem Solving Workshop**.

EXAMPLE 5 Prove that triangles are congruent

Write a proof.



GIVEN ▶ $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

PROVE ▶ $\triangle ACD \cong \triangle CAB$

Plan for Proof

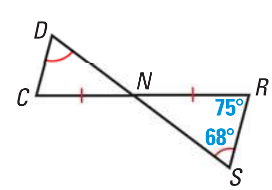
a. Use the Reflexive Property to show that $\overline{AC} \cong \overline{AC}$.

b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	1. Given
a.	2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property of Congruence
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	3. Given
b.	4. $\angle B \cong \angle D$	4. Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	5. Definition of $\cong$ $\triangle$s

✓ GUIDED PRACTICE for Examples 4 and 5



4. In the diagram, what is $m\angle DCN$?

5. By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$?

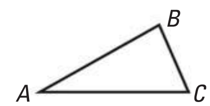
PROPERTIES OF CONGRUENT TRIANGLES The properties of congruence that are true for segments and angles are also true for triangles.

THEOREM *For Your Notebook*

THEOREM 4.4 Properties of Congruent Triangles

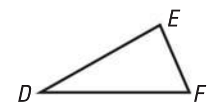
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong \triangle ABC$.



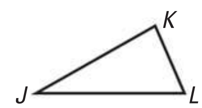
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



4.2 EXERCISES

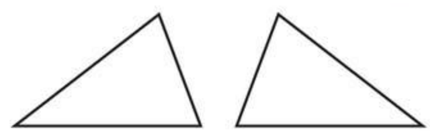
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 21, 24, 27, and 30

SKILL PRACTICE

1. **VOCABULARY** Copy the congruent triangles shown. Then label the vertices of the triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.

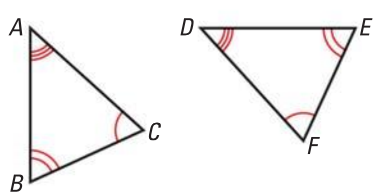


2. ★ **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? *Explain.*

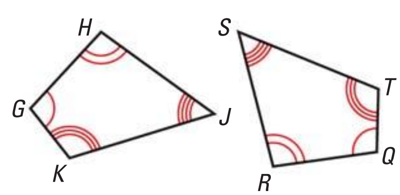
EXAMPLE 1 on p. 225 for Exs. 3–4

USING CONGRUENCE Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures.

3. $\triangle ABC \cong \triangle DEF$

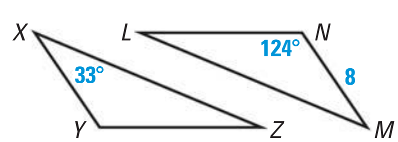


4. $GHJK \cong QRST$



EXAMPLE 2 on p. 226 for Exs. 5–10

READING A DIAGRAM In the diagram, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

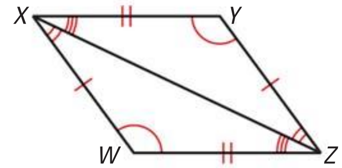


5. $m\angle Y =$ _?_
6. $m\angle M =$ _?_
7. $YX =$ _?_
8. $\overline{YZ} \cong$ _?_
9. $\triangle LNM \cong$ _?_
10. $\triangle YXZ \cong$ _?_

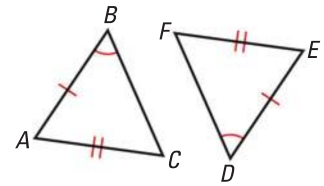
EXAMPLE 3
on p. 226
for Exs. 11–14

NAMING CONGRUENT FIGURES **Write a congruence statement for any figures that can be proved congruent. *Explain* your reasoning.**

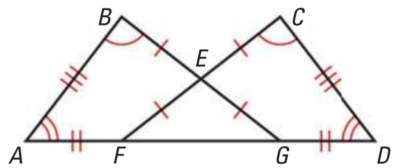
11.



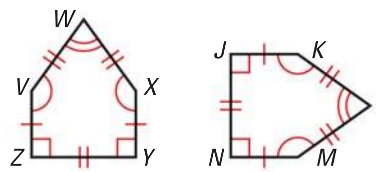
12.



13.



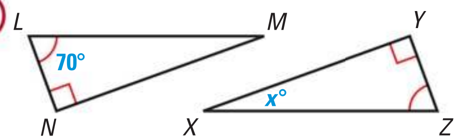
14.



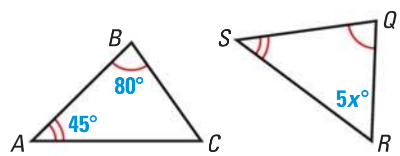
EXAMPLE 4
on p. 227
for Exs. 15–16

THIRD ANGLES THEOREM **Find the value of *x*.**

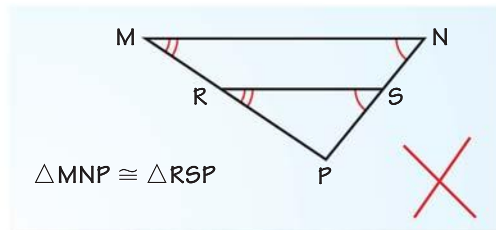
15.



16.



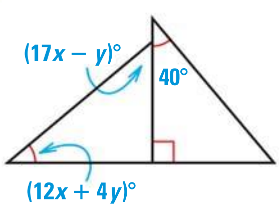
17. **ERROR ANALYSIS** A student says that $\triangle MNP \cong \triangle RSP$ because the corresponding angles of the triangles are congruent. *Describe* the error in this statement.



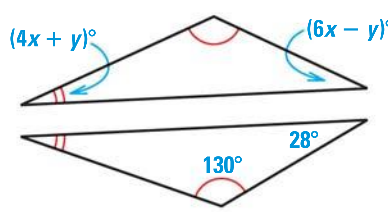
18. ★ **OPEN-ENDED MATH** Graph the triangle with vertices $L(3, 1)$, $M(8, 1)$, and $N(8, 8)$. Then graph a triangle congruent to $\triangle LMN$.

xy ALGEBRA **Find the values of *x* and *y*.**

19.



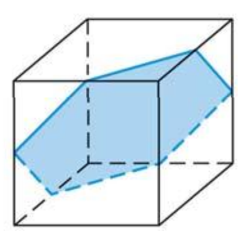
20.



21. ★ **MULTIPLE CHOICE** Suppose $\triangle ABC \cong \triangle EFD$, $\triangle EFD \cong \triangle GIH$, $m\angle A = 90°$, and $m\angle F = 20°$. What is $m\angle H$?

Ⓐ 20° Ⓑ 70° Ⓒ 90° Ⓓ Cannot be determined

22. **CHALLENGE** A hexagon is contained in a cube, as shown. Each vertex of the hexagon lies on the midpoint of an edge of the cube. This hexagon is equiangular. *Explain* why it is also regular.



PROBLEM SOLVING

23. RUG DESIGNS The rug design is made of congruent triangles. One triangular shape is used to make all of the triangles in the design. Which property guarantees that all the triangles are congruent?

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24. ★ OPEN-ENDED MATH Create a design for a rug made with congruent triangles that is different from the one in the photo above.

(25.) CAR STEREO A car stereo fits into a space in your dashboard. You want to buy a new car stereo, and it must fit in the existing space. What measurements need to be the same in order for the new stereo to be congruent to the old one?

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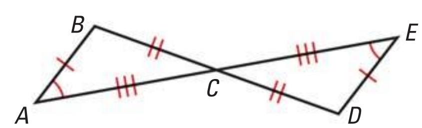
EXAMPLE 5 on p. 227 for Ex. 26

26. PROOF Copy and complete the proof.

GIVEN ▶ $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$

PROVE ▶ $\triangle ABC \cong \triangle EDC$



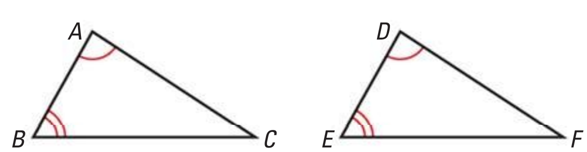
STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$	1. Given
2. $\angle BCA \cong \angle DCE$	2. ___?___
3. ___?___	3. Third Angles Theorem
4. $\triangle ABC \cong \triangle EDC$	4. ___?___

27. ★ SHORT RESPONSE Suppose $\triangle ABC \cong \triangle DCB$, and the triangles share vertices at points B and C. Draw a figure that illustrates this situation. Is $\overline{AC} \parallel \overline{BD}$? *Explain.*

28. PROVING THEOREM 4.3 Use the plan to prove the Third Angles Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle B \cong \angle E$

PROVE ▶ $\angle C \cong \angle F$



Plan for Proof Use the Triangle Sum Theorem to show that the sums of the angle measures are equal. Then use substitution to show $\angle C \cong \angle F$.

◯ = WORKED-OUT SOLUTIONS on p. WS1

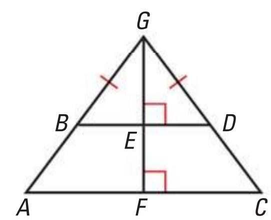
★ = STANDARDIZED TEST PRACTICE

29. **REASONING** Given that $\triangle AFC \cong \triangle DFE$, must F be the midpoint of $\overline{AD}$ and $\overline{EC}$? Include a drawing with your answer.

30. ★ **SHORT RESPONSE** You have a set of tiles that come in two different shapes, as shown. You can put two of the triangular tiles together to make a quadrilateral that is the same size and shape as the quadrilateral tile.

Explain how you can find all of the angle measures of each tile by measuring only two angles.

31. **MULTI-STEP PROBLEM** In the diagram, quadrilateral $ABEF \cong$ quadrilateral $CDEF$.



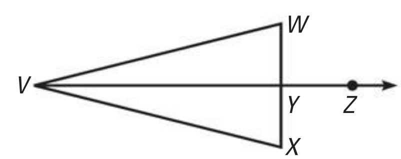
a. *Explain* how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.

b. *Explain* how you know that $\angle GBE \cong \angle GDE$.

c. *Explain* how you know that $\angle GEB \cong \angle GED$.

d. Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? *Explain*.

32. **CHALLENGE** Use the diagram to write a proof.



GIVEN ▶ $\overline{WX} \perp \overrightarrow{VZ}$ at Y, Y is the midpoint of $\overline{WX}$, $\overline{VW} \cong \overline{VX}$, and $\overrightarrow{VZ}$ bisects $\angle WVX$.

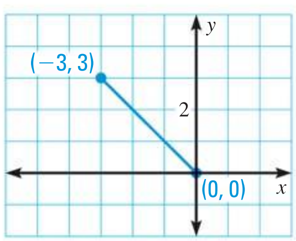
PROVE ▶ $\triangle VWY \cong \triangle VXY$

MIXED REVIEW

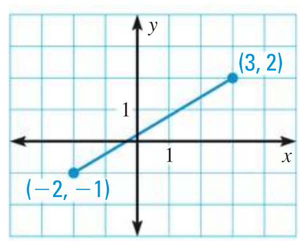
PREVIEW
Prepare for Lesson 4.3 in Exs. 33–35.

Use the Distance Formula to find the length of the segment. Round your answer to the nearest tenth of a unit. *(p. 15)*

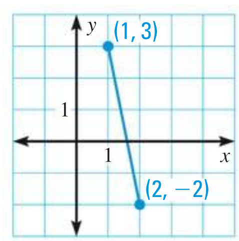
33.



34.

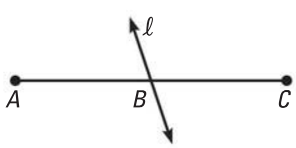


35.

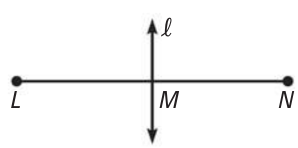


Line ℓ bisects the segment. Write a congruence statement. *(p. 15)*

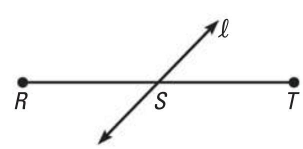
36.



37.



38.



Write the converse of the statement. *(p. 79)*

39. If three points are coplanar, then they lie in the same plane.

40. If the sky is cloudy, then it is raining outside.